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On the artificial freezing of sands

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ABSTRACT: A numerical and experimental study of the artificial freezing of saturated sand is presented. The numerical part of the work summarizes the main characteristics of the finite element approach adopted for solving the non linear heat transfer problem and the technique that introduces the latent heat effects during the water/ice phase transition. The experimental investigation was undertaken in order to calibrate the developed program and to choose a suitable procedure for determining the thermal properties of soil. To this purpose, a series of laboratory tests was carried out on large sample of saturated sand the temperature of which was brought well below the freezing point through the circulation of liquid nitrogen within a metal pipe.

1 INTRODUCTION

One of the techniques which can be adopted for improving the mechanical characteristics of saturated soils consists in their artificial freezing through the circulation of low temperature fluids within pipes inserted in the ground [Andersland et al., 1989]. The complex, non linear, nature of the freezing process suggests the use of numerical methods for the solution of a variety of problems of practical interest.

It should be observed, however, that the numerical analysis of soil freezing represents a non trivial problem due to the various mechanical and thermal phenomena involved. Among them are: the non linear conductive transfer of heat within the geotechnical medium; the water/ice phase transition and the consequent latent heat effects; the influence of the seepage flow of water on the convective heat transfer; the volume increase of the frozen soil and the possible migration of water toward the transition zone; the change of the mechanical properties of the frozen soil, and in particular of its shear strength and time dependent creep properties, with respect to those of the natural soil; etc.

In order to reduce the complexity of the solution algorithm, here only the thermal aspects of the problem have been considered and it was assumed that: the soil is perfectly saturated; its thermal properties do not depend on the stress and strain changes; the effects of convection and of the possible seepage flow of water can be ignored; the migration of water towards the phase change zone is neglected.

The first and second assumptions seem reasonable and should not appreciably limit the range of application of the numerical technique. Also the third assumption can be accepted since the presence of a significant seepage flow should be avoided in order to permit an effective freezing treatment. Note that this assumption rules out the movement of fluid from the problem at hand and permits to use the same governing equation for the heat transfer both above and below the freezing temperature.

The validity of the fourth hypothesis strongly depends on the characteristics of soil, and it can be accepted mainly for coarse textured soils, such as sands.

On the basis of the above assumptions, the analysis of freezing is reduced to the solution of the so called Stefan problem [Carslaw and Jaeger, 1959], i.e. of the transfer of heat by conduction accounting for phase changes.

In the following, the governing equations and the finite element formulation are first illustrated. Some details are also provided on the technique and on the procedure adopted to account for the latent heat during the water/ice transition. Then, the results of some laboratory freezing tests on large size cylindrical samples of sand are presented. On their basis, the computer program has been checked and a procedure was chosen for determining the thermal properties of the soil.

2. GOVERNING EQUATION AND BOUNDARY CONDITIONS

The equation of continuity, governing the two dimensional flow of heat through an isotropic solid, can be written in the following form [Carslaw and Jaeger, 1959]

$$\frac{\partial}{\partial x}(K \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(K \frac{\partial T}{\partial y}) + \bar{Q} = \rho c \frac{\partial T}{\partial t} \quad , \qquad (1)$$

where x and y are the Cartesian axes; T and t denote, respectively, temperature and time; K is the coefficient of thermal conductivity; ρ and c are the mass density and the thermal capacity, or specific heat, and $\bar{Q}$ is the rate of internal flow generation per unit volume.

As to the boundary conditions, let consider first the situation in which no phase change occurs. In this case four main types of conditions can be imposed on different portions of the boundary Γ of the flow domain Ω.

The temperature is known on Γ_1, whilst the heat flow component in the direction normal to the boundary is prescribed on Γ_2

$$T(x,y,t) - \bar{T} = 0 \quad \text{on } \Gamma_1 \quad , \qquad (2a)$$

$$K \,(i_x \frac{\partial T}{\partial x} + i_y \frac{\partial T}{\partial y}) - \bar{q}_n = 0 \quad \text{on } \Gamma_2 \quad . \qquad (2b)$$

In eqs.(2) $\bar{q}_n$ and $\bar{T}$ represent, respectively, the imposed rate of heat flow per unit area and the imposed temperature; i_x and i_y are the direction cosines of the unit outward vector normal to the boundary. Note that the material parameters and the coefficients characterizing the boundary conditions depend, in general, on the temperature.

The convection and radiation boundary conditions are not discussed here since they will not be considered in the subsequent parts of the paper.

In the case of phase transition a surface F(x,y,t)=0 exists, having an a priori unknown shape, that separates liquid and solid phases. Similarly to other moving boundary problems [Crank, 1984], two different conditions, related respectively to the temperature and to its space derivatives, characterize this surface. The first one imposes that the temperatures of the frozen T_f and unfrozen T_u phases facing the moving surface are equal to the melting/freezing temperature T_o.

$$T_f(x,y,t) = T_u(x,y,t) = T_o \quad . \qquad (3a)$$

The second one represents the relationship between the net rate of heat flow in the direction normal to the moving surface, the latent heat per unit volume and the velocity v_n of the surface itself

$$K_f \,(i_x \frac{\partial T_f}{\partial x} + i_y \frac{\partial T_f}{\partial y}) + $$

$$- K_u(i_x \frac{\partial T_u}{\partial x} + i_y \frac{\partial T_u}{\partial y}) = L \cdot \rho \cdot v_n \quad . \qquad (3b)$$

In eq.(3b) K_f and K_u are the coefficients of thermal conductivity of the frozen and unfrozen phases, respectively; L is the latent heat per unit mass and ρ is the mass density.

3. FINITE ELEMENT FORMULATION

The finite element formulation of heat conduction problems without phase changes will be recalled first. The heat flow domain Ω is subdivided into elements in which the temperature depends on the nodal values, for the e-th element:

$$T_e(x,y,t) = \underline{b}_e^T(x,y) \; \underline{T}_e(t) \quad , \qquad (4)$$

where T_e is the temperature within the element and vectors $\underline{T}_e$ and $\underline{b}_e$ collects, respectively, the nodal temperatures and the interpolation functions.

By writing eq.(1) in the so called "weak form", and by considering the natural boundary conditions (eqs.2b), mathematical manipulations lead to the following matrix relationship for the entire mesh,

$$\underline{M}\,\underline{T} + \underline{N}\,\dot{\underline{T}} = \bar{\underline{f}}_\Omega + \bar{\underline{f}}_\Gamma \quad . \qquad (5)$$

Matrices and vectors in eq.(5) are obtained by suitably overlapping the contributions of each element, denoted by the index e,

$$\underline{M}_e = \int_{\Omega_e} (\nabla \underline{b}_e^T)^T \; \underline{K}_e \; (\nabla \underline{b}_e^T) \; d\Omega \quad , \qquad (6a)$$

$$\underline{N}_e = \int_{\Omega_e} \underline{b}_e \; (\rho c)_e \; \underline{b}_e^T \; d\Omega \quad , \qquad (6b)$$

$$\bar{\underline{f}}_{e\Omega} = \int_{\Omega_e} \underline{b}_e \; \bar{Q}_e \; d\Omega \quad , \qquad (7a)$$

$$\bar{\underline{f}}_{e\Gamma} = \int_{\Gamma_{e2}} \underline{b}_e \; \bar{q}_{en} \; d\Gamma \quad . \qquad (7b)$$

Here $\nabla^T = \{\partial/\partial x \, , \, \partial/\partial y\}$ is the linear differential operator, $\underline{K}_e$ is the 2x2 matrix of the thermal conductivity coefficients and $\underline{i}_e^T = \{i_x \, , \, i_y\}$ is the direction cosine vector (cf.eq.2b). Vectors $\bar{\underline{f}}_{e\Omega}$ and $\bar{\underline{f}}_{e\Gamma}$ collect, respectively, the nodal heat flows due to internal heat generation per unit volume and those associated with the imposed rate of heat flow on Γ_{e2}.

Before integrating along time the system of scalar equations (5) it is necessary to impose the essential boundary conditions expressed by eq.(2a).

The step-by-step time integration of eq. (5) was based on the assumption of linear variation of the time dependent variables within a time increment $\Delta t>0$. This leads to an iterative process for each time increment which terminates when the difference between the nodal temperatures at $t+\Delta t$ evaluated at two subsequent iterations decreases below a given lower limit.

4. PHASE TRANSITION

The transition between solid and liquid phases is introduced in the solution of Stefan problem by the boundary conditions (3) on the surface separating the frozen and unfrozen zones, which is also referred to as "moving boundary".

Some of the finite element approaches presented in the literature, which can be referred to as "variable mesh" methods [Zabaras and Ruan, 1990], modify the geometry of the grid during the iterative solution process until a part of its boundary approximates with the required accuracy the shape of the moving surface at a chosen time. They are usually accurate, but their use becomes rather complex if several moving surfaces exist which join with each other when freezing propagates.

An alternative class of approaches, the so called "fixed mesh" techniques [Voller et al., 1990], operate on grid of constant geometry and allow the moving boundary to pass through the elements. Usually they are less accurate than the preceding ones, but offer the non negligible advantage of handling several simultaneous moving surfaces without requiring particular programming provisions.

Some of them introduce the latent heat effects through a variation of the heat capacity c within a given temperature range. Other techniques [Morgan et al., 1978] adopt the enthalpy (i.e. the integral of the relationship between ρc and T) as a new variable.

A technique different from the above was adopted in this study [Gioda et al.,1993]. The latent heat is viewed as a positive or negative source of heat, uniformly distributed on the volume, which becomes active when the freezing/ melting temperature is reached, and that is accounted for in the finite element analysis in integral form.

At the beginning of calculations the nodal vector of total latent heat is evaluated by the following equation, where the summation runs over the elements,

$$\underline{q}_L = \Sigma_e \int_{\Omega_e} \underline{b}_e \, L\rho \, d\Omega \quad . \tag{8}$$

When the temperature of the n-th node reaches the phase transition value, the node is "constrained" so that no further variation of its temperature may occur. The increments of nodal heat Δq_{nj} are then evaluated for each (j-th) time step, until the accumulated value reaches the previously evaluated nodal latent heat,

$$\Sigma_j \, \Delta q_{nj} = q_{nL} \quad . \tag{9}$$

When eq.(9) is fulfilled the node is released and its temperature is again allowed to vary with time.

This technique is apparently more "robust" than those based on the fictitious variation of the heat capacity or on the enthalpy approach. In fact, it does not show numerical instability due to the latent heat effects and relatively large steps can be adopted for time integration. On the other hand, it involves perhaps a larger computer time due to the matrix manipulations required for constraining the nodes that reach the freezing temperature.

5. EXPERIMENTAL INVESTIGATION

A series of laboratory freezing tests was performed, on which basis it was possible to check the computer program and to choose a suitable procedure for determining the equivalent thermal parameters of saturated sand [Gioda et al., 1993].

The equipment consists of a thermally insulated cylinder (fig.1), having internal diameter and height of 1 m, that before the test is filled with sand with uniform density. A steel pipe installed at the centre of the container is used for the circulation of liquid nitrogen.

The cylindrical geometry of the container and its insulation, consisting of polyurethane foam, lead to axisymmetric and planar heat flow conditions during the tests. This permits an easier interpretation of the experimental results by the finite element analysis.

Before the deposition of sand, which grain distribution is shown in fig.2, about 10 cm of water were placed in the container. Then deposition was initiated by spreading the sand out from a sieve which was kept at a constant level above the water surface. Additional water was introduced in the container during this process, so that the water level was always about 10 cm above the deposed sand. This technique lead to a saturated soil having a fairly uniform density throughout the container.

The same technique was adopted for preparing small size samples the density of which was subsequently determined. Their water content ranged between 0.279 and 0.30, the dry unit weight between 14.3 and 14.6 kN/m^3 and the porosity was between 0.417 and 0.436.

A small gap of about 2 cm was left between the sand and the upper part of the container in order to avoid possible damages caused by the volume increase of water during freezing. Drainage was allowed by three small diameter pipes. Owing to some impurity, the freezing temperature of water was -0.5°C.

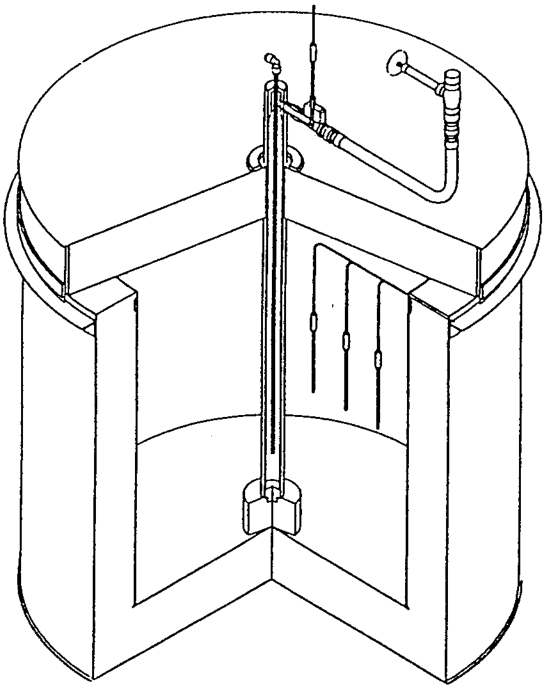

Fig. 1 Equipment for the laboratory freezing tests.

During the deposition of sand nine thermal transducers were placed at mid height of the container. They were located along three radii, at an angle of 120° from each other, and at a distance of 150 mm, 270 mm and 390 mm from the axis.

Three additional transducers were used to measure the temperatures of the room, of the nitrogen at its outlet from the steel pipe and of the outer part of the insulation. They operate from about -200°C up to +500°C, with an accuracy higher than 0.5°C. Two thermocouples were also introduced within the polyurethane foam in order to check the effectiveness of the thermal insulation.

The transducers were connected to a personal computer governed by and ad hoc developed software for data acquisition.

Three tests have been performed using pipes with external diameter equal to 33.7 mm, 60.3 mm and 88.9 mm, respectively. These are commercially available pipes that can be adopted in actual freezing treatments.

The initial temperature of the soil ranged between 18°C and 19°C. In order to facilitate the subsequent interpretation of the experimental data, the nitrogen flux was regulated in such a way that a constant temperature (-183°C) was measured

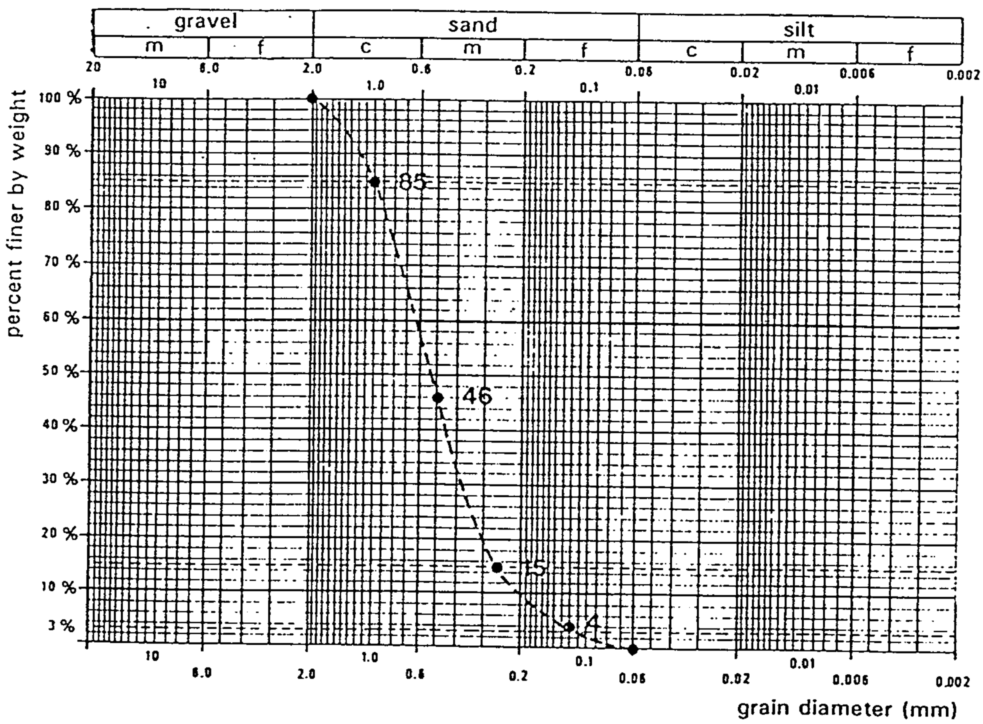

Fig. 2 Grain size distribution of the sand used in the freezing tests.

for all tests at the pipe outlet.

The variation with time of the readings of the thermal transducers for the first test is shown in fig.3. Diagrams (a), (b) and (c), representing the temperatures within the sand, were obtained by averaging the readings of the transducers at the same distance from the axis. The maximum difference between the temperatures measured at the same distance from the pipe was less than 1.5 °C.

The remaining two curves in fig.3 represent the room temperature (d) and the temperature of the outer part of the insulating material (e). It can be observed that the polyurethane foam provided an adequate thermal insulation since its temperature is influenced by the room temperature, but it is barely affected by the temperature inside the container.

The coolant flux was stopped when the farthest transducers from the axis reached 0°C. After some hours the temperature of all transducers reached the same value (about -1°C) which remained practically constant for several days, until the equipment was dismounted.

6. ANALYSIS OF THE EXPERIMENTAL RESULTS

The geometry and boundary conditions of the tests allow to base their numerical interpretation on a plane (normal to the cylinder axis) and axisymmetric scheme. A one dimensional finite element mesh was adopted consisting of a "line" of four nodes quadrilateral elements. All the mesh sides are impervious to heat, but for the side representing the freezing pipe which temperature coincided with that of the coolant outlet (-183°C).

A major problem consists in determining the values of the thermal parameters characterizing the homogeneous medium equivalent to the saturated sand. They are the thermal conductivity k [W/(m K)], the thermal capacity c [J/(kg K)] and the latent heat per unit mass L [J/kg].

While the thermal parameters of water are known with a relatively high degree of accuracy, those of the dry sand are more uncertain, owing also to the marked influence of the mineralogic characteristics. An average value for the conductivity of sand can be c_{sand} = 710 [J/(kg K)].

The thermal properties of the homogeneous "equivalent" medium can be evaluated according to Kersten [1949]. In particular, the thermal capacity depends on those of the mineral constituting the sand and of the water (or ice), and on the water content w. Hence, the equivalent thermal capacities below and above freezing are expressed as

$$c_{frozen} = c_{sand} + w\, c_{ice} \quad , \qquad (10a)$$

$$c_{unfrozen} = c_{sand} + w\, c_{water} \quad . \qquad (10b)$$

In the above relationships, c_{sand} is the thermal capacity of the dry sand which was assumed equal to the above mentioned average value.

The equivalent latent heat (per unit mass) L_{eq} is defined as the product of the latent heat of the water L by the water content w

$$L_{eq} = L\, w \quad . \qquad (11)$$

The evaluation of the equivalent thermal conductivity k was based on the results presented by Johansen and Frivik [1980]. It depends on the conductivity of the water (or ice), on that of quartz k_q and on that of the remaining fraction of sand k_r through the following expressions which hold, respectively, for temperatures below and above freezing

$$k_{frozen} = k_{ice}^{n} \left(k_q^{q} \;\; k_r^{(1-q)} \right)^{(1-n)} , \qquad (12a)$$

$$k_{unfrozen} = k_{water}^{n} \left(k_q^{q} \;\; k_r^{(1-q)} \right)^{(1-n)} . \qquad (12b)$$

In using eqs.(12), where q is the content of quartz and n the porosity of the sand, the following values of the relevant parameters were adopted,

$$k_q = 7.7 \text{ [W/(m K)]} \quad ;$$

$$k_r = 2.5 \text{ [W/(m K)]} \quad ; \quad q = 50\% \quad .$$

It was also taken into account that the thermal conductivity of ice undergoes a non negligible variation when the temperature decreases well below the freezing point. To account for this, the following linear interpolation was adopted of the experimental data presented by Frivik and Thorbergsen [1980],

$$k_{ice}\text{[W/(m K)]} = 2.3 - 0.015\ T\ [°C] \quad . \qquad (13)$$

Eqs.(10-13) have been introduced in the finite element simulation of the freezing tests, obtaining the results shown by dashed lines in fig.3.

Note that the stepwise pattern of the numerical diagrams in the vicinity of the phase change temperature depends on the particular procedure adopted for taking into account the latent heat effects.

7. CONCLUSIONS

A study of the non linear conduction of heat within saturated sands has been presented, with particular reference to the

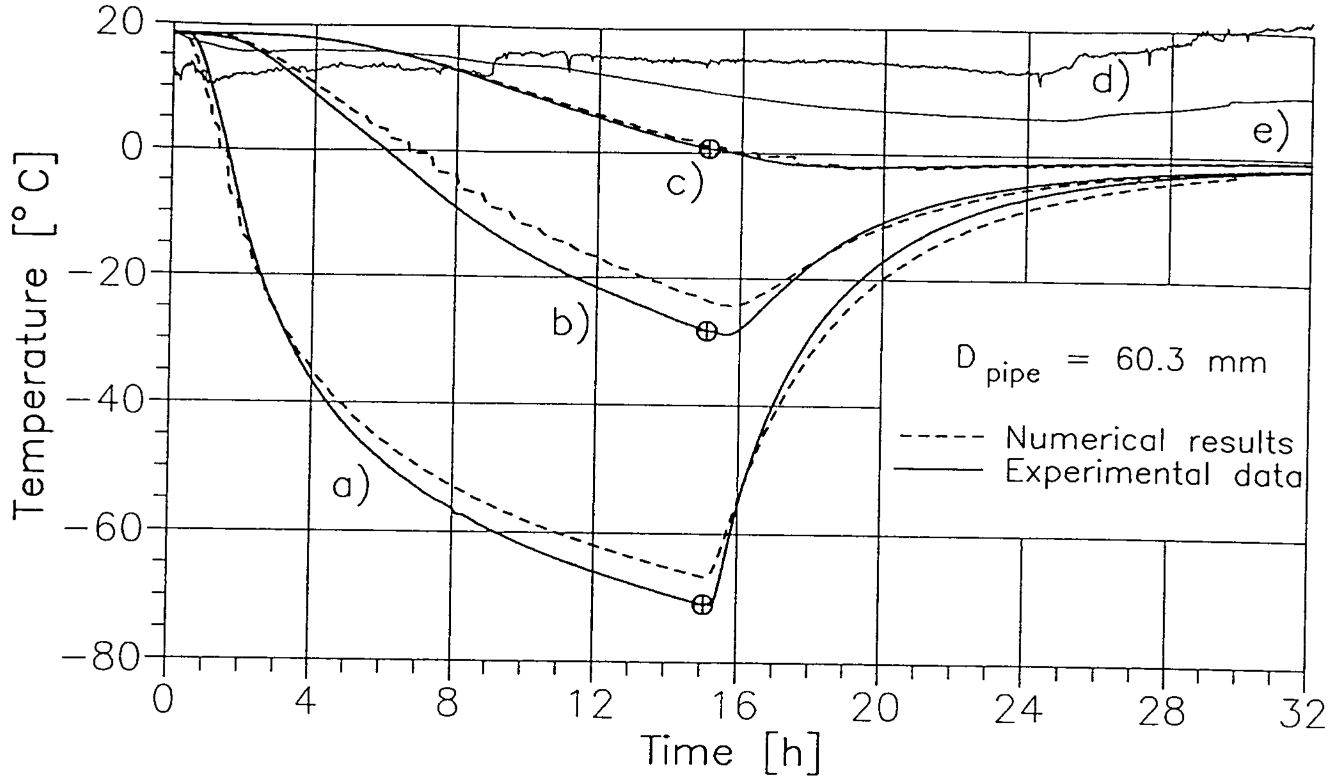

Fig. 3 Comparison between experimental and numerical results for a freezing test.

finite element analysis of the artificial freezing of soils.

The experimental data of a laboratory freezing test have been compared with the corresponding numerical results, in terms of the progress of the frozen boundary with time. The reasonable agreement between the two set of results indicates that the developed numerical procedure is able to solve heat transfer problems related to ground freezing with an acceptable accuracy from an engineering view point.

Possible extensions of this study seem possible concerning, in particular, the "optimization" of some of the geometrical and thermal parameters characterizing the artificial freezing treatments, such as the distance and diameter of the freezing pipes and the temperature of the fluid.

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